Math Virtual Learning

Precalculus with Trigonometry

May 4, 2020

Precalculus with Trigonometry
Lesson: May 4th, 2020

Objective/Learning Target:
Students will be able to find exact values of a trigonometric expression using the half-angle formulas.

Let's Get Started:

Watch the video below to see how the half angle formulas are developed from the double angle formulas in the previous two lessons.

Watch Video: Derive Trig Half Angle Identities

Watch from the beginning for good introduction to the half angle formulas for sine, cosine, and tangent.

Recall these formulas while watching the video.

Double Angle Identities

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

*** Please note that tangent has more than 1 formula.

Half-Angle Identities

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1+\cos A}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1-\cos A}{\sin A}$$

Alternatives for Sine & Cosine

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Finding Exact Values

Now that you've seen where the formulas come from, how can they be used?

Watch the video below to see how these formulas can be used to find the exact values of certain angles. You only have to watch till **3:15**, however I would suggest watching the whole video to get a good idea of how we will be using these formulas in the next couple days.

Video: Half Angle Formulas - How to Use

Example 1:

Earlier, you were asked to find the value of $\tan\frac{3\pi}{8}$ without a calculator.

$\frac{3\pi}{8} = \frac{1}{2}\cdot\frac{3\pi}{4}$ so we can use the formula $tan\frac{a}{2} = \frac{\sin a}{1+\cos a}$ for $a = \frac{3\pi}{4}$

$$\tan\frac{3\pi}{8} = \frac{\sin\frac{3\pi}{4}}{1+\cos\frac{3\pi}{4}}$$
$$= \frac{\frac{\sqrt{2}}{2}}{1+\frac{-\sqrt{2}}{2}}$$

If we simplify this expression, we get $\sqrt{2}+1$.

Example 2:

Find the exact value of $\cos\left(-\frac{5\pi}{8}\right)$.

$-\frac{5\pi}{8}$ is in the 3^{rd} quadrant.

$$-\frac{5\pi}{8} = \frac{1}{2}\left(-\frac{5\pi}{4}\right) \rightarrow \cos\frac{1}{2}\left(-\frac{5\pi}{4}\right) = -\sqrt{\frac{1+\cos(-\frac{5\pi}{4})}{2}}$$

$$= -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{1}{2}\cdot\frac{2-\sqrt{2}}{2}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

Example 3:

Given the function $\cos a = \frac{4}{7}$ and $0 \leq a < \frac{\pi}{2}$, find $\tan \frac{a}{2}$.

You can use either $\tan \frac{a}{2}$ formula.

$$\tan \frac{a}{2} = \frac{1 - \frac{4}{7}}{\frac{\sqrt{33}}{7}} = \frac{3}{7} \cdot \frac{7}{\sqrt{33}} = \frac{3}{\sqrt{33}} = \frac{\sqrt{33}}{11}$$

Practice

On a separate piece of paper, use the Half-Angle & Double-Angle Identities to determine the exact values for each of the following situations.

1. $\sin 105^\circ$
2. $\tan \dfrac{\pi}{8}$
3. $\cos \dfrac{5\pi}{12}$
4. $\cos 165^\circ$
5. $\sin \dfrac{3\pi}{8}$
6. $\tan\left(-\dfrac{\pi}{12}\right)$
7. $\sin \dfrac{11\pi}{8}$
8. $\cos \dfrac{19\pi}{12}$

The $\cos a = \dfrac{5}{13}$ and $\dfrac{3\pi}{2} \le a < 2\pi$. Find:

9. $\sin 2a$
10. $\cos \dfrac{a}{2}$
11. $\tan \dfrac{a}{2}$
12. $\cos 2a$

The $\sin a = \dfrac{8}{11}$ and $\dfrac{\pi}{2} \le a < \pi$. Find:

13. $\tan 2a$
14. $\sin \dfrac{a}{2}$
15. $\cos \dfrac{a}{2}$
16. $\sin 2a$

Some of these problems are double angle problems that are good review of the previous lessons. For the purposes of this lesson, you can skip those, but they are good review and it is suggested that you do those as well.

Practice - **ANSWERS**

1. $-\dfrac{\sqrt{2+\sqrt{3}}}{2}$

2. $\sqrt{2}-1$

3. $\dfrac{\sqrt{2-\sqrt{3}}}{2}$

4. $-\dfrac{\sqrt{2+\sqrt{3}}}{2}$

5. $\dfrac{\sqrt{1+\sqrt{2}}}{2}$

6. $-2-\sqrt{3}$

7. $-\dfrac{\sqrt{1+\sqrt{2}}}{2}$

8. $\dfrac{\sqrt{2-\sqrt{3}}}{2}$

9. $-\dfrac{120}{169}$

10. $-\sqrt{\dfrac{9}{13}}$

11. $-\dfrac{2}{3}$

12. $-\dfrac{119}{169}$

13. $\dfrac{16\sqrt{57}}{7}$

14. $\sqrt{\dfrac{11-\sqrt{57}}{22}}$

15. $\sqrt{\dfrac{11+\sqrt{57}}{22}}$

16. $-\dfrac{16\sqrt{57}}{121}$

Additional Resource Videos:

Evaluating for sine using the half angle formula

Learn how to evaluate using the half angle formula of sine

Using the half angle of cosine to evaluate cos

How to evaluate for the half angle of tangent

Additional Practice:

Double & Half Angle Identities - KUTA (with solutions)

Do Problems 5-8, 9, 11, 15, 18-21, and 24

Double & Half Angle Formulas Practice (with solutions)

Do Problems 7-12